ADI KAVI NANNIAH UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc.Mathematics

IV-SEMESTER

M401: Measure Theory

[W.E.F.2016 A.B.]

(Model Question Paper)

Time: 3 Hours **Max. Marks: 75**

Answer **ALL** questions. Each question carries 15 marks **Marks:** $5 \times 15 = 75$

1. Define a measurable space, give an example and verify that it is a measurable space. Prove that, if $A \subset B \in \mathcal{B}$, then $\mu(A) \le \mu(B)$ where μ is the measure on X.

(OR)

2. State and prove Fatou's Lemma

3. State and prove Lebesgue Convergence Theorem

(OR)

4. State and prove Hahn Decomposition Theorem

5. State and prove Radon – Nikodym Theorem.

(OR)

6. Prove that (a) $\mathcal{B}$ of μ^* - measurable sets is a σ - algebra. and (b) $\bar{\mu}$ is μ^* restricted to $\mathcal{B}$, then $\bar{\mu}$ is a complete measure on $\mathcal{B}$
7. State and prove State Caratheodory theorem

(OR)

8. State and prove Riesz representation theorem.

9. Answer any **THREE** questions of the following

    a. Define the signed measure, positive set, and distinguish between the null set and a set of measure zero through an example
    b. Prove that the countable union of positive sets is positive
    c. Define Caratheodory outer measure and Haussdorff measure
    d. State Fubini theorem, product measure and define cross section of a set E.

---------*---------

12/12/17

ADIKAVI NANNAYA UNIVERSITY
SEMISTER END EXAMINATIONS
M.Sc.Mathematics
IV- SEMESTER
M402 : NUMERICAL ANALYSIS
[W.E.F.2016 A.B.]
(Model Question Paper)

Time: 3 Hours **Max. Marks: 75**

Answer **ALL**questions. Each question carries 15 marks. **Marks** : 5 X 15=75

1) Find a root of the equation $\cos x - xe^x = 0$ by using regula- falsi method

(OR)

2) By using Muller method find the smallest positive root of the equation $f(x) \equiv x^3 - 5x + 1 = 0$.

3) Find the inverse of the matrix

$$\begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix}$$

Using partition method. Hence, solve the system of equations $AX = b$, where $b = [-10, 8, 7, -5]^T$.

(OR)

4) Solve the system of equations

$$2x_1 - x_2 = 7$$
$$-x_1 + 2x_2 - x_3 = 1 \quad \text{Using Gauss – Seidel method.}$$
$$-x_2 + 2x_3 = 1$$

5) Derive Bessel formula and find the value of $g(0.25)$ given that

x	:	0.1	0.2	0.3	0.4	0.5
$g(x)$	:	9.9833	4.9667	3.2836	2.4339	1.9177

(OR)

6) Obtain the piecewise quadratic interpolating polynomials for the function $f(x)$ defined by the data:

x	:	-3	-2	-1	1	3	6	7
$f(x)$	:	369	222	171	165	207	990	1779

Find and approximate value of $f(-2.5)$ and $f(6.5)$

Dr. T. HYMAVATHI
Convener-P.G. B.O.S.
Applied Mathematics & Mathematics
Adikavi Nannaya University
Rajamahendravaram-533 2..5

7) Evaluate the integral $I=\int_1^2 \frac{2x\,dx}{1+x^4}$, using the Gauss – Legendre 1-point, 2-point and 3-point quadrature rules. Compare with the exact solution $I=\tan^{-1}(4)-(\pi/4)$.

(OR)

8) Solve the initial value problem $u'=-2tu^2, u(0)=1$ with $h=0.2$ on the interval [0, 0.4]. Use the fourth order classical Runge – Kutta method. Compare with the exact solution.

9) Answer any <u>THREE</u> of the following.

a) Find the root of the equation $f(x)\equiv x^3-5x+1=0$

b) Perform two iterations of the Chebyshev method to find an approximate value of 1/7. Take the initial approximation as $x_0=0.1$

c) Solve the equations $x_1+x_2+x_3=6,\ \ 3x_1+3x_2+4x_3=20,\ \ 2x_1+x_2+3x_3=13$ using the Gauss elimination method.

d) Derive the formula for the first derivative of $y=f(x)$ of $O(h^2)$ using central difference approximations.

e) Find singlestep method for the differential equation $y'=f(t,y)$, which produce exact results for $y(t)=a+be^{-t}$

***

Dr. T. HYMAVATHI
Convener-PG B.O.S.
Applied Mathematics & Mathematics
Adikavi Nannaya University
Rajamahendravaram-533 296

ADIKAVI NANNAYA UNIVERSITY
SEMESTER END EXAMINATIONS
M.Sc. MATHEMATICS
IV SEMESTER
M(403) GRAPH THEORY
(w.e.f. 2016 A.B.)
MODEL QUESTION PAPER

Time: 3Hours | Max. Marks: 75

Answer ***ALL*** Questions and Each Question Carries 15 Marks | Marks: 5 X 15 = 75

1. (a) Prove that the number of vertices of odd degree in a graph is always even
   (b) Explain Konigsberg Bridge problem

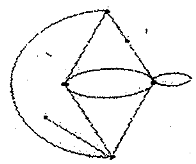
(OR)

2. (a) Prove the following statement:
   In a connected graph G, any minimal set of edges containing at least one branch of every spanning tree of G is a cut set
   (b) Show that the ring sum of any two cut-sets in a graph is either a third cut-set or an edge – disjoint union of cut-sets

3. Prove that " The Complete graph of five vertices is non-planar

(OR)

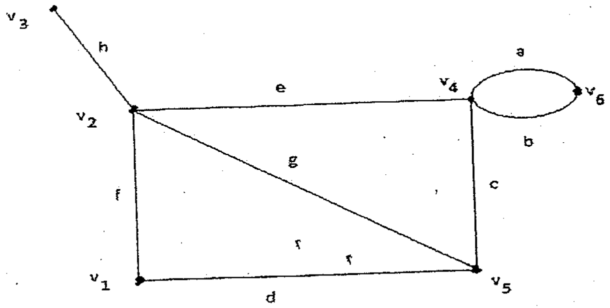
4. Obtain the Dual of the following graph



5.(a) If A(G) is an incidence matrix of a connected graph G with a vertices, then show that the rank of A(G) is n-1
   (b)If B is a circuit matrix of connected graph G with e edges and n-vertices then prove that Rank of B = e-n+1

(OR)

6. Let A and B be the respective circuit matrix and the incidence matrix of a self loop free graph whose columns are arranged using the same order of edges. Then every row of B is orthogonal to every row of A. i.e., $A.B^T = B.A^T = 0$ (Mod 2). Verify the result for the following graph



Dr. T. HYMAVATHI
Convener-P G B O S.
Applied Mathematics & Mathematics
Adikavi Nannaya University
Rajamahendravaram-533 296

7. Show that the vertices of every planar graph can be properly colored with five colors

(OR)

8. State and prove Max-flow-min –cut theorem

9. Answer any THREE of the following

(a) Define Euler and Hamiltonian graph and give one example for each.

(b) Define Tree, Spanning Tree and give one example for each.

(c) Show that Kuratowski second graph is non planar

(d) Show that every tree with two or more vertices is 2-chromatic.

(e) Define fundamental circuit matrix

ADIKAVI NANNAYA UNIVERSITY
SEMESTER END EXAMINATIONS
M.Sc. Mathematics
IV-SEMESTER
M404-LINEAR PROGRAMMING
(W.E.F.2016A.B)
Model Question Paper

Time:3 hrs **Max.Mks:75**

Answer **ALL** questions. Each question carries 15 Marks. **Marks : 5 X 15=75**

1) Solve the following LP problem graphically
Maximize $z = 8000x_1 + 7000x_2$,
Subject to $3x_1 + x_2 \leq 66, x_1 + x_2 \leq 45, x_1 \leq 20, x_2 \leq 40$ and $x_1, x_2 \geq 0$

(OR)

2) Solve the LP problem:
Maximize $z = 3x_1 + 2x_2 + 5x_3$,
Subject to the constraints
$x_1 + 2x_2 + x_3 \leq 430, 3x_1 + 2x_3 \leq 460$,
$x_1 + 4x_2 \leq 420$ and $x_1, x_2, x_3 \geq 0$

3) Use Big-M method to solve the problem
Maximize $z = 6x_1 + 4x_2$
Subject to the constraints
$2x_1 + 3x_2 \leq 30, 3x_1 + 2x_3 \leq 24$,
$x_1 + x_2 \geq 3$ and $x_1, x_2 \geq 0$. Is the solution unique? If not, give two different solutions.

(OR)

4) Apply the principle of duality to solve the LP problem
Maximize $z = 3x_1 - 2x_2$
Subject to the constraints $x_1 + x_2 \leq 5, x_1 \leq 4, 1 \leq x_2 \leq 6$ and $x_1, x_2 \geq 0$.

5) A car hire company has one car at each of five depots a,b,c,d and e. A customer requires a car in each town, namely A,B,C,D and E. Distance (in kms) between depots (origins) and towns (destinations) are given in the following distance matrix:

	a	b	c	d	e
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

How should cars be assigned so as to minimize the distance travelled.

(OR)

6) Solve the travelling -salesman problem given by the following data
$C_{12} = 20, C_{13} = 4, C_{14} = 10, C_{23} = 5, C_{34} = 6, C_{25} = 10, C_{35} = 6, C_{45} = 20$,
where $C_{ij} = C_{ji}$ and there is no route between cities i and j if the value for C_{ij} is not shown.

7) A Steel company has three open hearth furnaces and five rolling mills. Transportation cost (Rupees per quintal) for shipping steel from furnaces to rolling are shown in the following table.

Mills / Furnaces	M_1	M_2	M_3	M_4	M_5	Capacities (in quintals)
F_1	4	2	3	2	6	8
F_2	5	4	5	2	1	12
F_3	6	5	4	7	3	14
Requirement (in quintals)	4	4	6	8	8	

What is the optimal shipping schedule?

(OR)

8) Solve the following transportation problem .

Cost-matrix

To / From				Available
	0	2	0	70
	10	4	0	30
	0	2	4	50
Required	70	50	30	

9) Answer any **THREE** of the following:

a) What do you mean by LPP? What are its limitations?

b) Write the steps used in the Simplex method.

c) Write the Mathematical formulation of Assignment problem

d) Write the dual of the following LP problem
Min. $Z = 3x_1 - 2x_2 + 4x_3$
Subject to the constraints:
$3x_1 + 5x_2 + 4x_3 \geq 7,\ 6x_1 + x_2 + 3x_3 \geq 4,\ 7x_1 - 2x_2 - x_3 \leq 10,$
and $x_1, x_2 \geq 0$

e) What is degeneracy problem in transportation Problems. What is its cause? How it can be Overcome.

****

ADIKAVI NANNAYA UNIVERSITY

M.Sc. Degree Examinations

Mathematics

IV-Semester

Paper - 405.1:DISCRETE DYNAMICAL SYSTEMS

(W.E.F. 2016 Admitted Batch)

(Model Question Paper)

Time : 3 Hours Max. Marks: 75

Answer ALL questions. Each question carries 15 marks

$(5 \times 15 = 75)$

(1) (a) Define fixed point, periodic point, attracting and repelling fixed points.

(b) Let f be a C' function and p be a fixed point of f such that $|f'(p)| < 1$. Show that there exists a neighborhood of p which is contained in $W^s(p)$.

(OR)

(2) Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function having a periodic point of period three. show that f has periodic points of all periods.

(3) (a) Define the Shift Map and show that the Shift map is continuous, it has 2^n periodic points of period n and there is an element with dense orbit

(b) Define Bifurcation, Saddle-node bifurcation, Pitch-fork bifurcation

(OR)

(4) (a) Let $f : X \to X$ be topologically transitive and suppose that the periodic points of f are dense in X. If X is infinite then f exhibits sensitive dependence on initial conditions

(b) Explain Period doubling bifurcation with an example

(5) Let D and E be metric spaces, $f : D \to D, g : E \to E$, and $\tau : D \to E$ be a topological conjugacy of f and g. Then, (i)$\tau^{-1} : E \to D$ is a topological conjugacy, (ii) $\tau \circ f^n = g^n \circ \tau$ for all natural numbers n, (iii) f is topologically transitive on D if and only if g is topologically transitive on E.

(OR)

(6) Suppose $p(x)$ is a polynomial If we allow cancellation of common factors in the expression of $N_p(x) = x - \dfrac{p(x)}{P'(x)}$, then $N_p(x)$ is always defined at roots of $p(x)$, a number is a fixed point of $N_p(x)$ if and only if it is a root of the polynomial, and all fixed points of $N_p(x)$ are attracting.

(7) (a) Show that all complex quadratic polynomials are topologically conjugate to a polynomial of the form $q_c(z) = z^2 + c$.

(b) Prove that the orbit of a complex number under iteration of a complex quadratic polynomial is either bounded or the number is in the stable set of infinity.

(OR)

(8) Let $f(z) = e^{\theta i}z$ and z_0 is a non zero complex number. Show that (a) z_0 is a periodic point of f if θ is a rational multiple of π, (b)if θ is not a rational multiple of π then z_0 is not a periodic point of f and its orbit is dense in the circle containing z_0.

(9) Answer ant THREE of the following

(a) Define Discrete Dynamical system and give three examples .

(b) Explain the concept of Phase Portrait with the help of an example

(c) Define Sensitive dependance, Devany Chaos and give an example of a dynamical system which is choatic in the sense of Devany

(d) Define Topological Transitivity and show that the existence of a dense orbit implies topological transitivity

(e) Define topological conjugacy and prove that the periodicity and period of a periodic point is is preserved by topological conjugacy

T. Hymavathi

Dr. T. HYMAVATHI
Convener-PG B.O.S.
Applied Mathematics & Mathematics
Adikavi Nannaya University
Rajamahendravaram-533 296

MODEL PAPER

M.Sc. DEGREE EXAMINATION

Mathematics

Fourth semester

(M405-27)-**OPERATOR THEORY**

(Effective from the admitted Batch 2016-2017)

Time:3hours

Max.Marks :75

Note: Answer ONE question from each unit. Each question carries 15 marks

UNIT - I

1. a) State and Prove Banach Fixed Point Theorem.

b) Suppose that $v(t)=x(t)-\mu\int_{a}^{t}\kappa(t,\tau)d\tau$ is continuous on $[a,b]$ and the kernel k is continuous on the triangular region R in the $t\tau$ plane given by $a\leq\tau\leq t,\quad a\leq t\leq b,$ then $v(t)$ equation has a unique solution.

(Or)

2. State and prove picard's Existence and Uniqueness theorem.

UNIT – II

3. a) Explain about Haar uniqueness theorem for best approximation.

b) Existence theorem for best approximation.

(Or)

4. a) Explain about Chebyshev Polynomials.

b) Explain about strict convexity

UNIT – III

5. State and Prove Spectral Mapping Theorem for polynomials.

(Or)

6. a) The Resolvent set $\rho(T)$ of a bounded linear operator T on a complex Banach space X is open; hence the spectrum $\sigma(T)$ is closed.

T. Hymavathi

Dr. T. HYMAVATHI
Convener-P.G. B.O.S.
Applied Mathematics & Mathematics
Adikavi Nannaya University
Rajamahendravaram-533 296

b) The Spectrum $\sigma(T)$ of a bounded linear operator $T: X \to X$ on a complex Banach space X is compact and lies in the disk given by

$$|\lambda| \leq \|T\|$$

UNIT – IV

7. Let $T: X \to Y$ be a linear operator. If T is compact, so is its adjoint operator $T^X : Y' \to X'$; here X and Y are normed spaces and X' and Y' the dual spaces of X and Y.

Or

8. Let $T: X \to X$ be a compact linear operator on a normed linear space X. then for every $\lambda \neq 0$ the range of $T_\lambda = T - \lambda I$ is closed

9. Answer any **three** of the Following questions :

a) Let $T: X \to Y$ be a mapping on a complete metric space X=(X, d), and suppose that T^m is a contraction on X for some positive integer then T has a unique fixed point.

b) Explain the definition of

i) External point ii) Haar condition iii) Normed space iv) Hilbert space

c) State and prove linear independence theorem

d) If X≠ $\{0\}$ is a complex Banach space and T $\in B(X, X)$ then $\sigma(T) \neq \phi$.

e) Let X and Y be Normed spaces and $T: X \to Y$ be a compact linear operator.Suppose that (x_n) in X is weakly convergent, say, $x_n \xrightarrow{w} x$.Then (Tx_n) is strongly convergent in Y and has the limit y = Tx.

ADI KAVI NANNIAH UNIVERSITY
SEMESTER END EXAMINATIONS
M.Sc.Mathematics
IV-SEMESTER
M405.3: Advanced Differential Equations
[W.E.F.2016 A.B.]
(Model Question Paper)

Time: 3 Hours **Max. Marks: 75**

Answer **ALL** questions. Each question carries 15 marks **Marks:** $5\times 15=75$

1. Let y and z be linearly independent solutions of $L(x)=(px')'+qx=0$ on (a,b) and let $A=p(t)\left[y(t)z'(t)-y'(t)z(t)\right]$. Define $G(t,s)=\begin{cases}-y(t)z(s)/A; t\le s\\ -y(s)z(t)/A; t>s\end{cases}$
Then $x(t)$ is a solution of the BVP $L(x)+f(t)=0; a\le t\le b, m_1x(a)+m_2x'(a)=0,$
$m_3x(b)+m_4x'(b)=0$ if and only if $x(t)=\int_a^b G(t,s)f(s)dx$

(OR)

2. The Green's function is given to be $G(t,s)=\begin{cases}-y(t)z(s)/A\ ; t\le s\\ -y(s)z(t)/A\ ; s\le t\end{cases}$, then prove that $x(t)$ is a solution of the BVP $L(x)+f(t)=0, a\le t\le b$ and $m_1x(a)+m_2x'(a)=0;$ $m_3x(b)+m_4x'(b)=0$ if and only if $x(t)=\int_a^b G(t,s)f(s)ds$

3. If $a'(t)$ exists and is continuous, then $x''+a(t)x'+b(t)x=0$ with $a(t),b(t)$ are real functions for $t\ge 0$ is oscillatory if and only if, the equation $x''+c(t)x=0$ is oscillatory with $c(t)=b(t)-\frac{a^2(t)}{4}-\frac{a'(t)}{2}$.

(OR)

4. State and prove Sturm's comparison theorem

5. If the matrix A of $x'=Ax$, $0\le t<\infty$ is having the characteristic roots with negative real parts, $B(t)$ is an $n\times n$ continuous matrix defined on $[0,\infty)$ is such that $\lim_{t\to\infty}\|B(t)\|=0$, then all solutions of $y'=Ay+B(t)y, 0\le t<\infty$ tend to zero as $t\to\infty$

(OR)

6. If the differential system $x'=f(t,x), x(t_0)=x_0, 0\le t_0\le t<\infty$ ...... (*) where x, x_0 and f are elements of R^n satisfying (i) $f(t,x)$ is continuous and satisfying Lipschitz's condition on the set
$\Delta=\{(t,x): t\ge 0, \|x\|<a<\infty\}$ and (ii) $f(t,0)=0, t\ge 0$, there exists a function $V(t,x)$ satisfying $V_t+V_x\cdot f\le g(t,V)$ and positive definite with the solution $y=0$ of $y'=g(t,y), y(t_0)=y_0>0$ is stable, then the solution $x=0$ of (*) is also stable.

T. Hymavathi
Dr. T. HYMAVATHI
Convener-P.G. B.O.S.
Applied Mathematics & Mathematics
...avi Nannaya Uni...
...avaram...

7. Prove that the solutions of ' $x'(t) = ax(t) + bx(t-r), 0 \le t_0 \le t < \infty$ with a, b real, $r > 0$, and $x(s) = \phi(s), t_0 - r \le x \le t_0$, with ϕ is real valued continuous function on $[t_0 - r, t_0]$ ' is bounded if $\int_{t_0-r}^{t_0} \phi^2(s)\,ds \le \infty, a \le 0$ and $|b| \le |a|$

(OR)

8. Prove that if $a > 0$ and $b < \frac{a}{e^a - 1}$, then the delay differential equation $x'(t) + ax(t) + bx([t]) = 0$ has no oscillatory solution

9. Answer any **THREE** questions of the following

a) Find the eigen values and eigen functions of $x'' + \lambda x = 0, 0 \le t \le \pi$ using $x(0) = x(\pi) = 0$

b) Prove that if $f(t,x)$ is defined on $[a,b]$ and $p = \frac{K(b-a)^2}{8} < 1$ with $|f(t,x) - f(t,y)| < K|x - y|$, then prove $\|x_{m+1} - x_m\| < p^m \|x_1 - x_0\|, m = 1, 2, \ldots$ where $\|x\| = \sup_{a \le t \le b} |x(t)|$ using induction.

c) Check the oscillations or non – oscillations of $x'' + e^t x = 0, t \ge 0$

d) What is fixed point technique and give an example.

e) Verify $y(t) = e^{-t}$ is a solution of $y'(t) + \frac{1}{e} y(t-1) = 0$

---------*--------

T. Hymavathi
Dr. T. HYMAVATHI
Convener-P.G. B.O.S.
Applied Mathematics & Mathematics
Adikavi Nannaya University
Rajamahendravaram-533 296